Examples of Markov Chains

repair breakdown

- A machine is either operating (1) or is broken (0) at the end of each shift, where $\boxed{P_{00} = .05}$, $\boxed{P_{01} = .95}$, $\boxed{P_{10} = .12}$, $\boxed{P_{11} = .88}$

P_{00} = conditional probability that machine is <u>broken</u> after next shift given that it is <u>broken</u> now,

P_{01} = conditional probability that machine is <u>operating</u> after next shift given that it is <u>broken</u> now,

P_{10} = conditional probability that machine is <u>broken</u> after next shift given that it is <u>operating</u> now,

P_{11} = conditional probability that machine is <u>operating</u> after next shift given that it is <u>operating</u> now

$$P = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{pmatrix} 0.05 & 0.95 \\ 0.12 & 0.88 \end{pmatrix} \end{array}$$

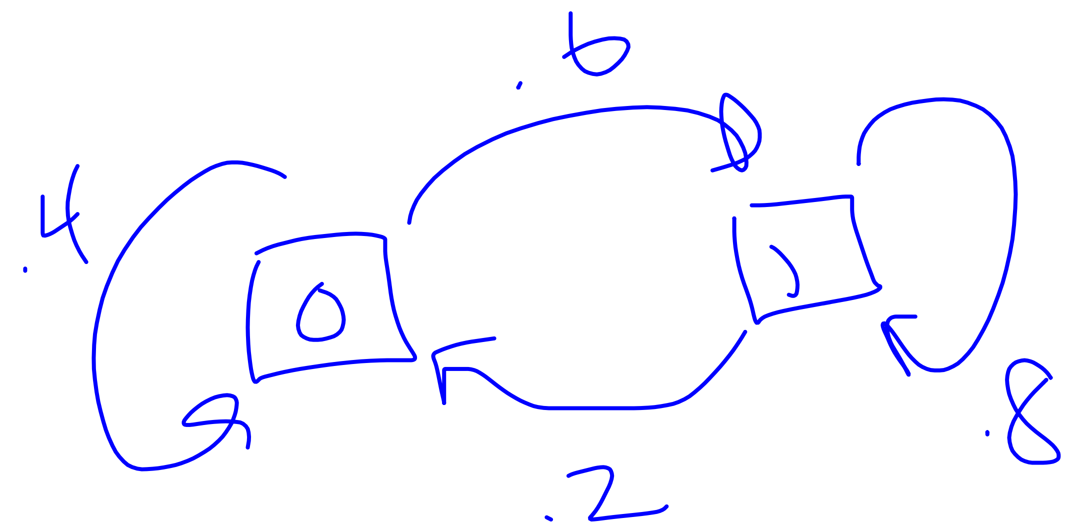
IN CLASS EXAMPLE.

EXAMPLE

Alice is taking a probability class and each week, she is either up-to-date or behind. If she is up-to-date in one week, the probability that she is up-to-date in the next week is 0.8. If she is behind in one week, the probability that she behind in the next week is 0.4.

Formulate this as a Markov chain. What are the states? What are the transition probabilities?

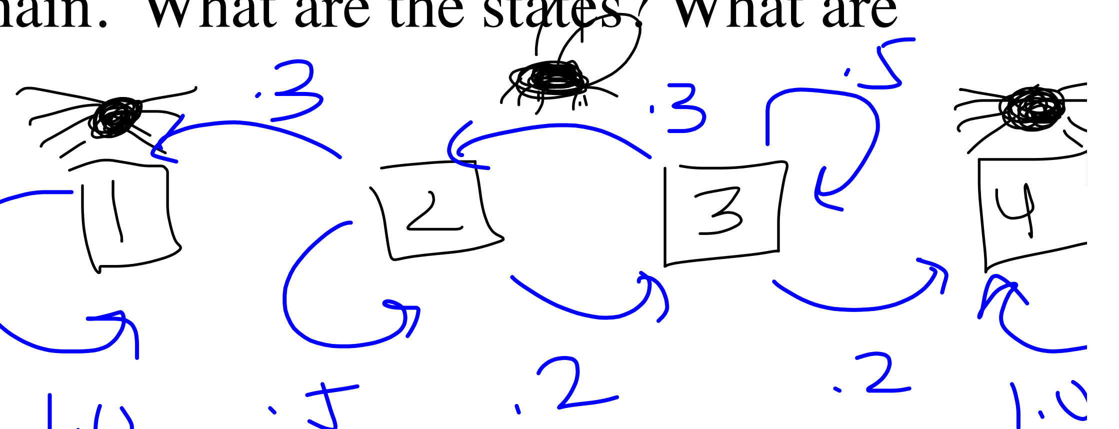
$$P = \begin{pmatrix} .4 & .6 \\ .2 & .8 \end{pmatrix}$$



EXAMPLE

A fly moves along a straight line in unit increments. At each time period, it moves one unit to the left with probability 0.3, one unit to the right with probability 0.2, and stays in place with probability 0.5. Two spiders are lurking in positions 1 and m; if the fly lands there, it is captured indefinitely.

Formulate this as a Markov chain. What are the states? What are the transition probabilities?



$$
P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ .3 & .5 & .2 & 0 \\ 0 & .3 & .5 & .2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{array}
$$

EXAMPLE:

Suppose that whether it rains today depends on previous weather conditions through the last three days (i.e., whether it rained or not). How many states are needed to model this as a Markov chain?

RRR

2^4 states

Suppose that if it has rained for the past three days, then it will rain today with probability 0.8. If it has not rained any the past three days, then it will rain today with probability 0.2. In any other case, the weather today will be, with probability 0.6, the same as the weather conditions yesterday.

What are the states?

What are the transition probabilities for this Markov chain?

$n=2$:

R N N

$n=0$ $n=1$ $n=2$

$n=3$:

N N R

N N N

$n=1$ $n=2$ $n=3$

.4

.6

$$P = \begin{array}{c|cccccccc}
 & RRR & RRN & RNR & RNN & NRR & NRN & NNR & NNN \\
\hline
RRR & .8 & .2 & 0 & 0 & 0 & 0 & 0 & 0 \\
RRN & 0 & 0 & .4 & .6 & 0 & 0 & 0 & 0 \\
RNR & 0 & 0 & 0 & 0 & .6 & .4 & 0 & 0 \\
RNN & 0 & 0 & 0 & 0 & 0 & 0 & .4 & .6 \\
NRR & .6 & .4 & 0 & 0 & 0 & 0 & 0 & 0 \\
NRN & 0 & 0 & .4 & .6 & 0 & 0 & 0 & 0 \\
NNR & 0 & 0 & 0 & 0 & .6 & .4 & 0 & 0 \\
NNN & 0 & 0 & 0 & 0 & 0 & 0 & .2 & .8
\end{array}$$